PERFORMANCE EVALUATION OF SYSTEM IDENTIFICATION ALGORITHMS for STRUCTURAL HEALTH MONITORING

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ABSTRACT:

In the last three decades several time domain methods for identifying dynamic characteristics of structures from measurements have appeared. Two of them had much attention and became the most popular methods in civil engineering applications for estimating the modal parameters: (1) A subspace approach for identifying a state space model (2) The Eigensystem Realization algorithm with Markov parameters computed from input/output data or output correlations when the input is not measured. The algorithms extract eigenvalues and eigenvectors on the premise that the system response is linear during the data collection. The system is assumed to be viscously damped and the excitation, when unknown, is assumed stationary and broadband. When the assumptions are satisfied and the noise in the measurements is white, exact results are obtained as the length of the data sequence approaches infinity (provided, of course, the system properties do not change during the measurements). In civil engineering applications System Identification algorithms are used to estimate the dynamic characteristics of structures using vibration (i.e. accelerogram) data. These dynamic characteristics, namely natural frequencies, damping ratios and mode shapes are fundamental properties for Structural Health Monitoring and are used for further analysis to identify possible damage in structures. In this study performance evaluation of the two system identification algorithms are presented using numerical study on a model of a 12-story reinforced concrete building instrumented with three uni-axial sensors at four floors. Moreover, the performance of the algorithms is further investigated using the benchmark data available from the Green Building at the Massachusetts Institute of Technology campus in Boston, MA. The Green Building has 21 floors and is instrumented with three uni-axial sensors at ten floors. Performance of the system identification algorithms is evaluated regarding the influence of duration. Based on the observations in the numerical study performed, 50Ts and 150Ts are recommended (within 5% limit in error coefficient) as the minimum duration of the output signal for modal frequency and mode shape identification, respectively (where Ts is fundamental period of the structure).

KEYWORDS: Eigensystem Realization Algorithm (ERA), Stochastic Subspace Identification Algorithm, Ambient Vibration, System Identification

1. INTRODUCTION

Vibration based structural health monitoring (SHM) has been widely used in civil structures, such as buildings, bridges, earthwork structures and industrial facilities. The purpose of SHM is to evaluate the condition of the structure and to identify damages. A variety of approaches have been presented in the literature for system identification and damage detection. The theory of vibration-based SHM is that the dynamic characteristics of a structure are a function of its physical properties. When there are changes in these physical properties, such as a decrease in stiffness due to localized structural damage, there will be corresponding changes in the dynamic characteristics. Doebling et al. (1996) presented a comprehensive review of vibration based SHM applications used in structures and buildings. Successful implementation of SHM depends on (1) measuring vibration data and

(2) data processing algorithms. The recent advancements in sensor technology make the custom designed accelerometers available for specific dynamical system/structure characteristics.

2. METHODOLOGY

System identification is an approach for using input-output data, and sometimes output data only, as is the case here, to build a mathematical model of a system. When the input is not available it is assumed to be stationary broad band noise. The price that is paid when the input is not available is that the recorded output must be of long duration and that the mode shapes cannot be scaled to mass. In civil engineering applications System Identification algorithms are used to estimate the dynamic characteristics of structures using vibration (i.e. accelerogram) data. These dynamic characteristics, namely natural frequencies, damping ratios and mode shapes are fundamental properties for the SHM and are used for further analysis to identify possible damage in structures. A typical schematic of System Identification algorithm for modal data estimation of the buildings is presented in Figure 1.

Figure 1 –A typical schematic of System Identification algorithm for modal data estimation

The goal of this work is comparing performance of system identification algorithms that estimate the dynamic characteristics of the building type of structures using ambient (output-only) vibration data. The input data of the algorithms is accelerometer measurements obtained from instrumented building. The output data of the algorithms is the dynamic parameters of the building, namely natural frequencies, damping ratios and mode shapes at sensor locations.

In the last three decades several time domain methods for identifying dynamic characteristics of structures from measurements have appeared. Two of them had much attention and became the most popular methods in civil engineering applications for estimating the modal parameters: (1) A subspace approach for identifying a state space model (2) The Eigensystem Realization algorithm with Markov parameters computed from input/output data or output covariance functions when the input is not available. Within the scope of this work these two algorithms are utilized for estimating the modal parameters. Brief descriptions of these algorithms are presented in the following.

Eigensystem Realization Algorithm (ERA):

ERA was proposed by Juang and Pappa (1985) and has been used for state-space system identification of aerospace structures, wind turbines, civil structures and many other systems. ERA generates a system realization in the time domain using matrices, known as Markov Parameters, that list the pulse responses connecting input and output coordinates. In the absence of inputs, correlation matrices between the output channels for different lags can replace the Markov Parameters. In the ERA algorithm the state-space model is obtained from a factorization of a Hankel matrix of Markov Parameters (or correlations) using the singular value decomposition.

It is assumed that the system that generates the data is of the form

$$x_{k+1} = Ax_k + \omega_k \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

where (w,n), referred to as the process and the measurement noise sequences, are assumed to be white, although they may be correlated. The assumptions on the stochastic signals and linearity lead to a covariance of the measured output that writes

$$E\left(y_{k+1}y_k^T\right) = \Lambda_{\mathrm{i}} = CA^{i-1}G \tag{3}$$

with

$$G = A\Sigma C^T + S \tag{4}$$

where $\Sigma = \mathrm{E}\left(\mathrm{x_k} x_k^T\right)$ and $S = E\left(\omega_k v_k^T\right)$. Equation 3 is pivotal as it shows that the output covariance, which can be estimated from the measurements, can be treated as Markov Parameters of a system having the triplet {A,G,C}. Accuracy in Covariance-Based-ERA (Cov_ERA) hinges on how well the values of Λ_{i}, estimated from time-limited data, approximate the asymptotic results as the duration approaches infinity. Once the covariance of the output is estimated the realization of the matrices in Equation 1 and Equation 2 is obtained using the Eigensystem Realization Algorithm (ERA) with a balanced partition of the singular values between Observability and Controllability.

Stochastic Subspace Identification Algorithm (N4SID):
Subspace Identification algorithms operate with projections of the data on its own past and are theoretically unified by the estimation of Kalman states. Subspace algorithms have been extensively studied in the past and a class of these algorithms is described in detail in Van Overschee and De Moor (1996). In this work we use the N4SID algorithm (Numerical algorithms for Subspace State Space System Identification). N4SID algorithm computes an estimate of the state sequence matrix and solves for the state space system matrices as the Least Squares solution of an overdetermined set of linear equations. Research efforts present quite successful results for these algorithms on estimation of modal parameters of civil structures (Lam and Meyel, 2011; Abdelghani et al., 1998).
The acronym N4SID stands for Numerical algorithms for Subspace State Space System IDentification. The stochastic algorithm that we implement here is identical to the Unweighted Principal Component (UPC) algorithm in the family of purely stochastic subspace schemes. In general, stochastic subspace algorithms operate by extracting Kalman filter estimates of the state trajectory from the data and computing the system matrices from these states.

3. SIMULATION DATA ANALYSIS: 12 STORY BUILDING

The system considered is a 12 story 3-D building model having the arrangement of lateral load resisting elements shown in Figure 2. The structure has 36 modes and has been assigned 5% damping in each of them. Since the system is symmetric in x-x (horizontal) but not in y-y there are 12 pure x-x translational modes and 24 modes that couple y-y translation with rotations of the floor plan.

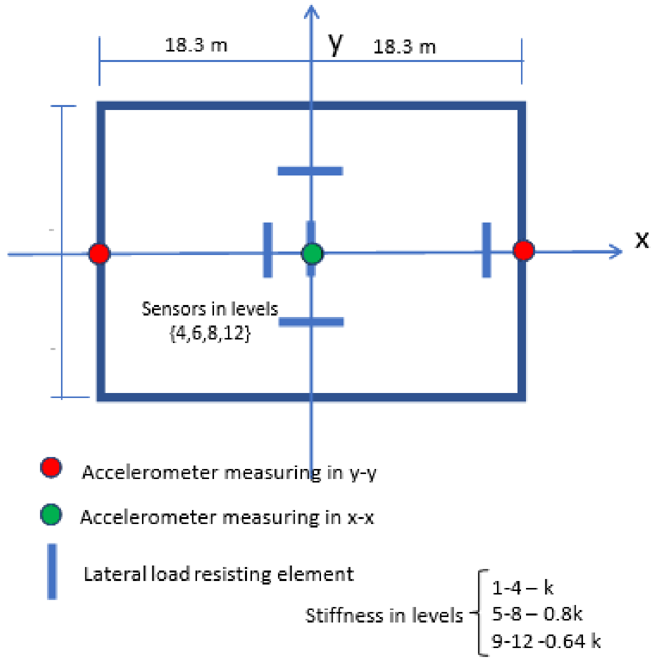

Figure 2 – Schematic plan of structure considered

Random generated excitation is applied at translational degree-of-freedoms. Excitation is filtered noise sampled at 50Hz with energy in the band 0-10 Hz. Of the total 36 modes 6 have frequencies that exceed 10Hz and are thus not excited. It is assumed that there are four x-direction accelerometers and eight y-direction ones all located in levels 4, 6, 8 and 12 as shown in the Figure 2. Independent white noise with 2% NSR (based on the x-x sensor in level #6) is added to all the channels. We process the data for the x-x and y-y together for both algorithms. The stabilization diagrams for covERA and N4SID Stabilization diagrams are depicted in Figure 3 and Figure 4 for frequencies up to 6Hz. There are 16 total modes in this bandwidth and it appears that 9 or 10 of them are sufficiently observable and controllable to be identified.

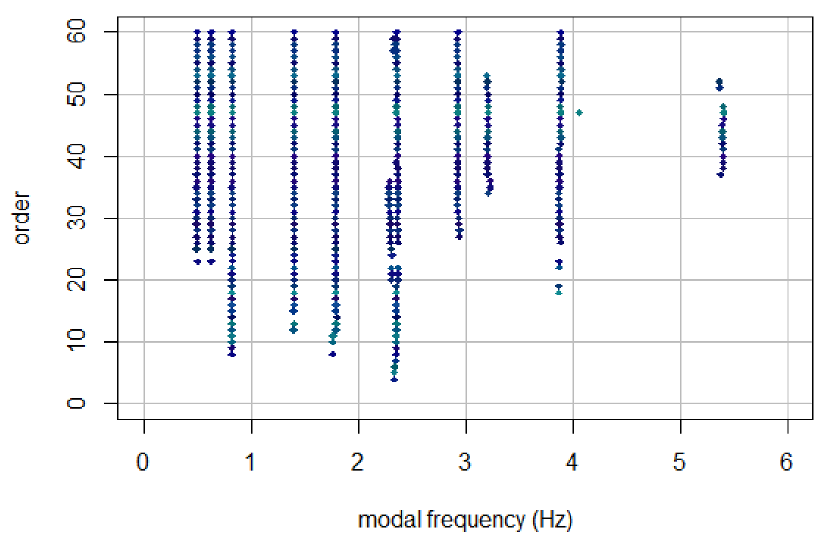

Figure 3 – Stabilization diagram for realizations obtained with covERA (ni=200) for x-x data.

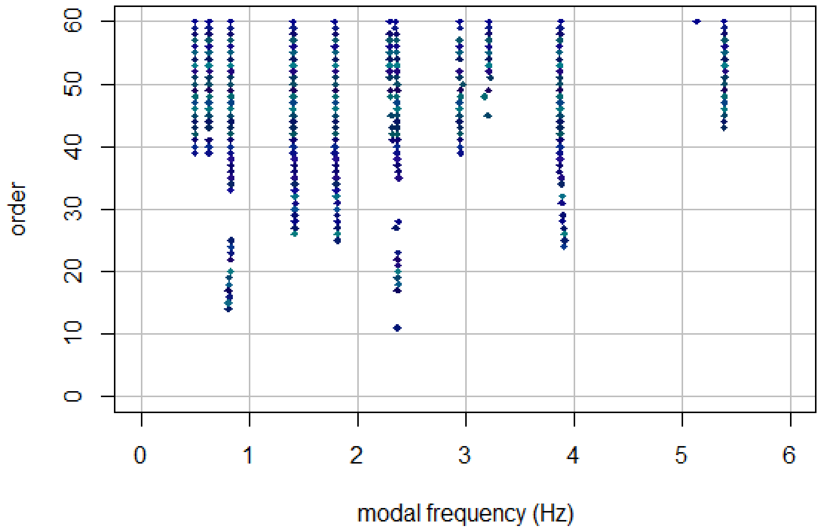

Figure 4 – Stabilization diagram for realizations obtained with N4SID (i=60)

A cursory inspection of the Figure 3 and Figure 4 show that there is good agreement between the two algorithms. The numerical estimates are compared with exact solution for frequencies and damping in Table 1.

Table 1 – Frequencies and damping comparisons

Mode #	Frequency (Hz)			Damping Ratio(%)		
	covERA	N4SID	Exact	covERA	N4SID	Exact
1	0.5010	0.5007	0.5000	5.3193	5.7787	5.00
2	0.6324	0.6321	0.6381	6.1960	6.3452	5.00
3	0.8287	0.8309	0.8373	4.9925	5.0394	5.00
4	1.4051	1.4150	1.4026	5.0622	4.8879	5.00
5	1.7899	1.8019	1.7899	5.4105	5.3553	5.00

The results of the numerical case study show that the two algorithms lead to quite consistent system identification models. Both algorithms identified user-selected five set of mode shape with corresponding undamped frequencies and damping ratios with close agreement with exact values.

Evaluation of the Data Duration

In order to evaluate the effect that the duration of the data has on the accuracy of the identification, a parametric study is performed. In the study, covERA modal identification is performed using a set of signal with varying duration. The results of the study are presented using the coefficients of error in the modal frequency and mode shape identification, which are defined as follows;

$$\epsilon_{fi} = 100\frac{|f_i - \hat{f_i}|}{|f_i|} \tag{5}$$

$$\epsilon_{\phi i} = 100 \frac{|\phi_i - \hat{\phi}_i|}{|\phi_i|} \tag{6}$$

where $f_i$and ϕ_i are the actual modal frequency and mode shape vector of the ith mode, respectively. $\hat{f}_i$and $\hat{\phi}$ are the estimated values. The results of this evaluation study for the first four modal frequencies and mode shapes are presented in Figure 5.

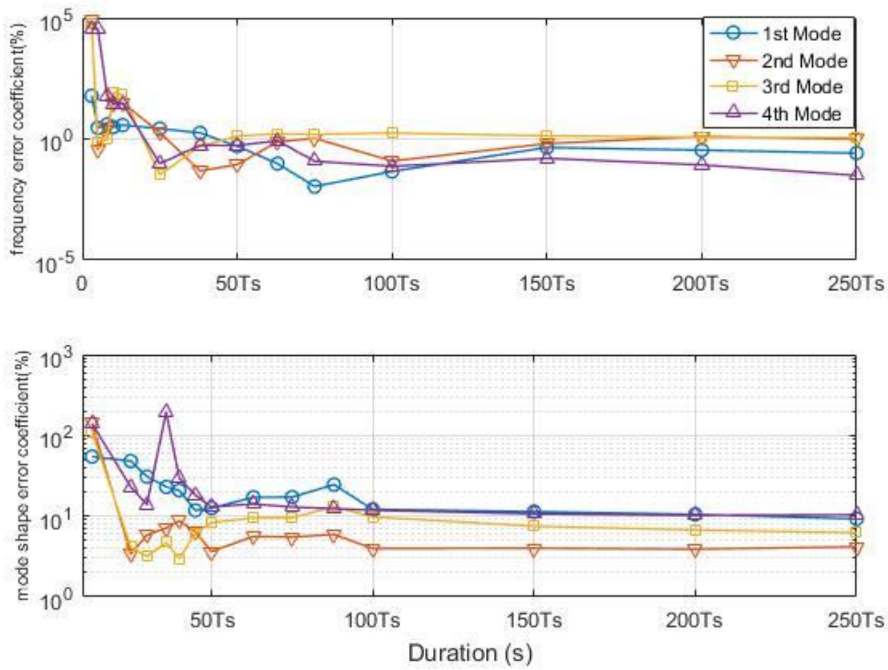

Figure 5 – Error Coefficients for Estimation of Frequency (top) and Mode Shapes (bottom) for varying data duration

As seen in Figure 5, identification accuracy quickly increases with increase in data duration. Data with duration below 50Ts leads to highly inaccurate estimates of mode shapes. It's observed that the accurate estimation of mode shapes requires longer data compare to frequency estimation; a result that is anticipated by the fact that the sensitivity of the measurements to the mode shapes is significantly smaller than to the frequencies. Within 5% limit in error coefficient, 50Ts and 150Ts (where Ts is fundamental period of the structure) are observed to be the minimum duration of the output signal for modal frequency and mode shape identification, respectively.

4. REAL DATA ANALYSIS: MIT GREEN BUILDING

The benchmark problem in the case study is obtained from a recent study by Sun and Buyukozturk (2018) on the Green Building located at the Massachusetts Institute of Technology campus in Boston, MA. The benchmark data and models are open to the public for algorithmic development and validation. The data and finite element models were provided by Dr. Sun per our request. The figures (Figure 6 and Figure 7) about the Green Building presented in this section are adopted from Sun and Buyukozturk (2018).

The Green Building has 21 stories above the ground (83.7 m) and a basement (3.8 m). The building was constructed as cast-in-place reinforced concrete and instrumented with 36 accelerometers to measure the building translational, torsional and vertical responses. The short and long directions of the Green Building are represented by North-South (NS) and East-West (EW), respectively.

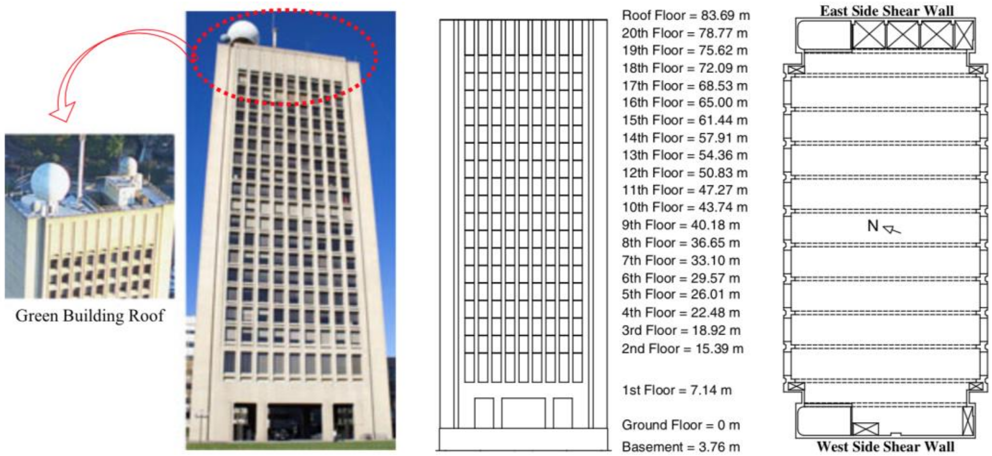

Figure 6 – MIT Green Building at MIT in Boston, MA (the east/west sides are concrete shear walls), Sun and Buyukozturk (2018)

As reported by Sun and Buyukozturk (2018), the Green Building is instrumented with 36 uniaxial EpiSensor ES-US2 force balance accelerometers designed by Kinemetrics Inc., CA, USA, by the United States Geological Survey. 34 sensors are deployed at different floors including the base and ground levels to record horizontal motions and four vertical sensors are placed at the base to record vertical motions. The detailed sensor locations and directions are illustrated in Figure 6. The sensors have a 24 bit digitizer with a recording range of ±4 g at a sampling rate of 200 Hz.

The accelerometers are installed below the floor slabs. At the floors instrumented, two sensors are measuring NS direction, and one sensor is measuring EW direction as shown in Figure 7.

The benchmark problem includes the detailed description of this building, 7 field measurement data sets (4 ambient data sets, 1 data set under an unidentified event, 1 data set under the excitation of fireworks, and 1 earthquake data set). Within the scope of this peer review study, only the first data set of the ambient data is used, namely D1, 0.

Taciroglu et al (2016) investigated the rocking behavior of the MIT Green Building and reported that sensors 7 and 8 are switched from original layout. This information brought uncertainty in these sensors and therefore in this case study we ignored the ground floor sensors and used only translational sensors located upper 9 floors (27 sensors). In this case we considered 9 sensors measuring EW directions and 18 sensors measuring at the floors 1, 2, 6, 7, 12, 13, 18, 19, 21.

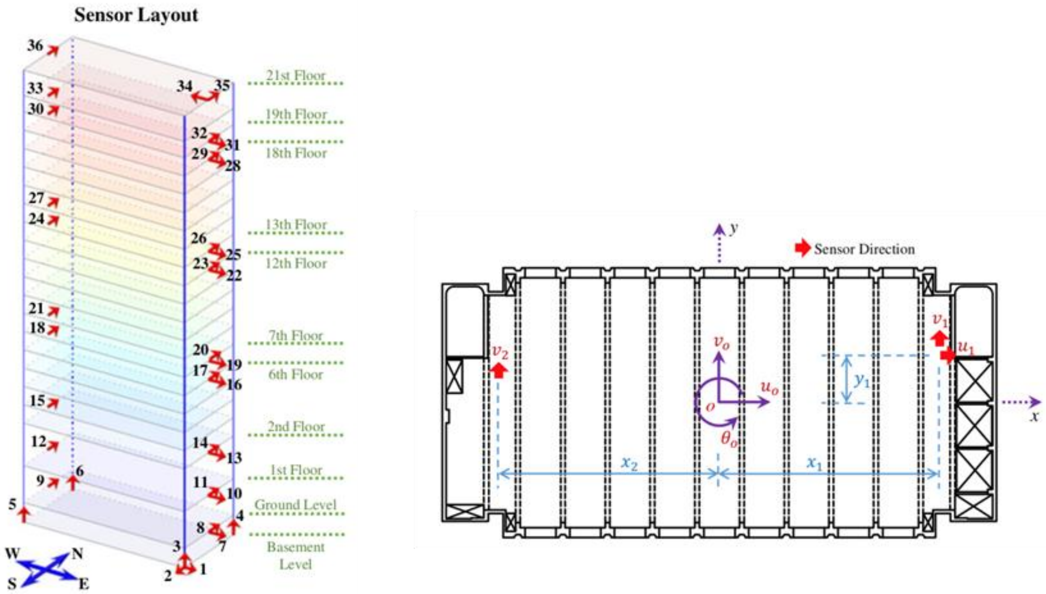

Figure 7 – Instrumentation of the Green Building, and Sensor Locations at a Typical Floor, Sun and Buyukozturk (2018)

In order perform a consistent and comparable study with Sun and Buyukozturk (2018), in the study we perform two separate identification process for two horizontal direction analyze:

1) The channels (9 sensors – one sensor per floor) in the E-W direction
2) The channels (18 sensors – 2 sensors per floor) in the N-S direction

Examination of the stabilizations diagrams shows that 7-8 of modes are sufficiently observable and controllable to be identified in EW direction and 5-6 of modes in NS direction. Identified frequencies are presented and compared with the Sun and Buyukozturk (2018) study in Table 2 and damping ratios for the identified modes are presented in Table 3.

Table 2 – Green Building Identified Fundamental Frequencies

Mode #	Frequency (Hz)					
	EW			NS		
	covERA	N4SID	Sun & Buyukozturk (2018)	covERA	N4SID	Sun & Buyukozturk (2018)
1	0.702	0.704	0.701	0.751	0.754	0.758
2	0.752	0.752		1.460	1.458	1.46 (Torsion)
3	1.448	1.462	1.46 (Torsion)	1.947		
4	2.109	2.082		2.815	2.819	2.814
5	2.516	2.532	2.545	3.571	3.578	
6	2.805			4.585	4.586	
7	4.589	4.622				5.017 (Torsion)
8	5.017	5.020	5.017(Torsion)	5.062		5.032
9			5.065			

Table 3 – Green Building Identified Damping Ratios

Mode #	Damping Ratio			
	EW		NS	
	covERA	N4SID	covERA	N4SID
1	0.025	0.026	0.014	0.019
2	0.011	0.012	0.007	0.007
3	0.011	0.019	0.060	
4	0.038	0.025	0.013	0.018
5	0.031	0.030	0.012	0.021
6	0.015		0.074	0.034
7	0.070	0.037		
8	0.013	0.009	0.013	

Identified mode shapes of East-West direction and North-South direction are compared in Figure 8 and Figure 9 respectively.

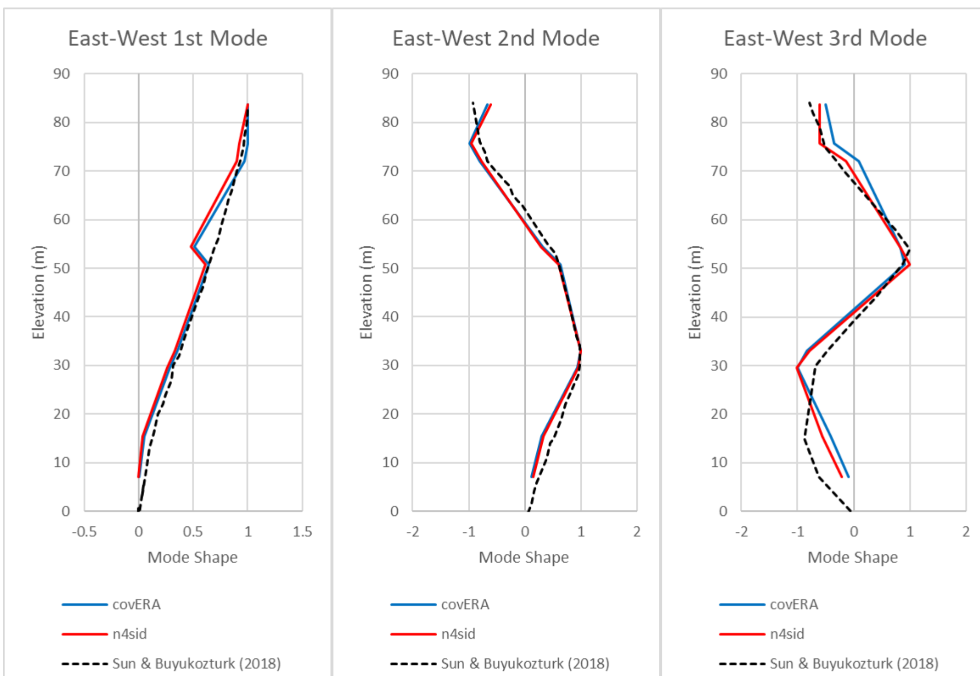

Figure 8 – Green Building East-West Mode Comparison

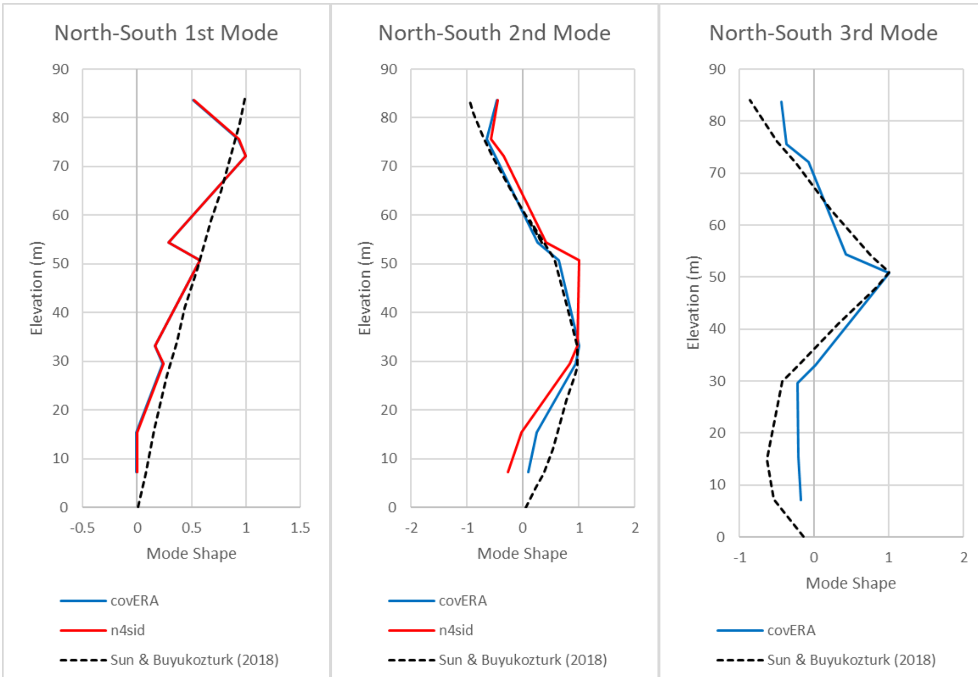

Figure 9 – Green Building North-South Mode Comparison

5. CONCLUDING REMARKS

This study reviews two system identification algorithms that are widely used in civil engineering applications. We restrict attention to the common case where vibrations arise from ambient sources so the input is not deterministically available. The algorithms are: 1) the Eigensystem Realization Algorithm based on output covariance and 2) A subspace approach often designated as N4SID. The focus of the study is on how the duration of the data affects the accuracy of the estimated modal parameters. The performance of both algorithms was found to be similar so selection between the two is a matter of personal preference or availability. Quantitative examination showed that the error in frequency becomes reasonably stable at durations of around 50 times the fundamental period, while the mode shapes took about 3 times longer to stabilize, requiring approximately 150 times the fundamental period. These values are recommended as minimum durations for performing stochastic system identification of buildings when using either of the algorithms considered.

6. ACKNOWLEDGEMENT

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REFERENCES

Abdelghani, M., Verhaegen, M., Van Overschee, P., & De Moor, B. (1998). Comparison study of subspace identification methods applied to flexible structures. *Mechanical systems and signal processing*, ***12*****(5)**, 679-692.

Crowley, H., & Pinho, R. (2010). Revisiting Eurocode 8 formulae for periods of vibration and their employment in linear seismic analysis. *Earthquake Engineering & Structural Dynamics*, ***39*****(2)**, 223-235.

Çelebi, M., Toksöz, N., & Büyüköztürk, O. (2014). Rocking behavior of an instrumented unique building on the MIT campus identified from ambient shaking data. *Earthquake Spectra*, ***30*****(2)**, 705-720.

Doebling, S. W., Farrar, C. R., Prime, M. B., & Shevitz, D. W. (1996). *Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review* (No. LA-13070-MS). Los Alamos National Lab., NM (United States).

Juang, J. N., & Pappa, R. S. (1985). An eigensystem realization algorithm for modal parameter identification and model reduction. *Journal of guidance, control, and dynamics*, ***8*****(5)**, 620-627.

Lam, X. B., & Mevel, L. (2011). Uncertainty quantification for eigensystem-realization-algorithm, a class of subspace system identification. *IFAC Proceedings Volumes*, ***44*****(1)**, 6529-6534.

Sun, H., & Büyüköztürk, O. (2018). The MIT Green Building benchmark problem for structural health monitoring of tall buildings. *Structural Control and Health Monitoring*, ***25*****(3)**, e2115.

Taciroglu, E., Çelebi, M., Ghahari, S. F., & Abazarsa, F. (2016). An investigation of soil-structure interaction effects observed at the MIT green building. *Earthquake Spectra*, ***32*****(4)**, 2425-2448.

Van Overschee, P., & De Moor, B. L. (2012). *Subspace identification for linear systems: Theory—Implementation—Applications*. Springer Science & Business Media.