Online Damage Detection Using Extended Kalman Filter

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Abstract

This paper focus on the detection of faults that take place abruptly and in situations where the time from the fault appearance to detection is important; in this setting the before and after strategy is inadequate. The general scheme for online detection uses a filter that represents the reference state and infers on the appearance of damage from inspection of the outputs. We examine a variant where appearance of the fault is inferred by inspecting the value of a parameter of a model of the healthy state that is estimated using Extended Kalman Filter (EKF) based augmented state approach. The selected parameter is not required to be the one where the damage takes place but can be a surrogate whose sensitivity to the damage is used for detection. A parameterization scheme is presented in order to obtain Jacobian of the state space models required in the EKF.

1 INTRODUCTION

Damage in structural systems is typically defined as changes that cause deterioration in some parameters that describe the stiffness. In damage detection the objective is to have a scheme that can detect, as early as possible, changes that may affect the performance of the system. In the "before and after" strategy the operating premise is that no damages take place during the data collection intervals and damage is identified from changes between two models. In this paper our focus is on the detection of damages that takes place abruptly and in situations where the time to detection is important. Online detection is typically done by formulating filters that represents the reference state and damage is inferred by inspecting the output of the filter as it processes the incoming measurements.

In this paper we introduce an online detection filter that detects damage by tracking the values of some parameter in a model. The key idea investigated is whether selection of the parameter to be tracked can be done without regard to whether it is in fact the parameter that is affected by the damage. To the best of the writer's knowledge, damage detection based on filters was first discussed by Mehra and Peshon (1971), who used the whiteness property of the Kalman filter innovation process as a feature. The seminal work on model based damage detection filters, where the objective is not just detection, but also isolation, i.e., identification of the specific nature of the fault, appears in (Beard 1971) and (Jones 1973). One of the first applications of filter based damage detection in structural engineering is due to Fritzen, et al. (1995), who used a bank of Kalman filters to detect damage.

The idea of appending the parameters to the state vector for their online identification is used in (Kopp and Orford 1964). Since the state estimation problem becomes nonlinear in this case, even if the system itself is linear, the extended Kalman filter is used here to perform the estimation. A fundamental contribution on the theory of the extended Kalman filter as a parameter estimator for linear systems is the work by Ljung (1979) who presented asymptotic behaviour of the filter. Panuska (1979) presented another form of the filter, where the state consists only of the parameters that are estimated and extended the work to the systems which are subjected to the correlated noise, (Panuska 1980). In recent years the EKF approach for parameter estimation has received significant attention in structural engineering with applications in damage detection appearing in (Yang et al. 2005, Liu et al. 2009).

2 EKF-BASED COMBINED STATE AND PARAMETER ESTIMATION

In this section we outline the EKF approach to the parameter estimation problem in the case where the system is linear and the nonlinearity arises from the augmentation of the state vector with unknown parameters. The system considered is assumed to have the following description

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) + Lw(t) \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

where $A(\theta) \in \mathbb{R}^{nxn}$, $B(\theta) \in \mathbb{R}^{nxr}$ and $L \in \mathbb{R}^{nxs}$ are the transition, input to state and process noise to state matrices, respectively and θ is a finite dimensional vector of parameters. $C \in \mathbb{R}^{mxn}$ is state to output matrix. $u(t) \in \mathbb{R}^{rx1}$, $x(t) \in \mathbb{R}^{nx1}$ and $y_k \in \mathbb{R}^{mx1}$ denote deterministic known input, state and measurement, respectively. The $w(t) \in \mathbb{R}^{sx1}$ is the process noise and v_k is the measurement noise. In the treatment here it is assumed that these are uncorrelated Gaussian stationary white noise sequences with zero mean and covariance of Q and R respectively. Additionally, it's also assumed that $w(t)$ and v_k are independent of θ. One begins by augmenting the state with the parameter vector $\theta = \theta(t)$, namely

$$z(t) \overset{def}{=} \begin{bmatrix} x(t) \\ \theta(t) \end{bmatrix} \tag{3}$$

The second step involves comprising a new state space model for the augmented state, namely

$$\dot{z}(t) = \bar{A}(\theta)z(t) + \bar{B}(\theta)u(t) + \bar{L}w(t) \tag{4}$$

where

$$\bar{A} = \begin{bmatrix} A(\theta) & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

$$\bar{B} = \begin{bmatrix} B(\theta) & 0 \end{bmatrix}^T \tag{6}$$

$$\bar{L} = \begin{bmatrix} L & I \end{bmatrix}^T \tag{7}$$

<u>Prediction Step:</u>

The a priori estimate of the state is obtained from

$$\dot{\hat{z}}(t) = \bar{A}(\theta)\hat{z}(t) + \bar{B}(\theta)u(t) \tag{8}$$

and we take $\hat{z}_k^- = \hat{z}(t)$. The a priori state error covariance is calculated from

$$\dot{\bar{P}}(t) = F(\hat{z}(t))\bar{P}(t) + \bar{P}(t)F(\hat{z}(t))^T + \bar{L}\bar{Q}\bar{L}^T \tag{9}$$

where

$$\bar{Q} = \begin{bmatrix} Q & [0] \\ [0] & q \end{bmatrix} \tag{10}$$

with q as the covariance of the pseudo noise introduced to drive the filter to change the estimate of θ. $F(\hat{z}(t))$ is the Jacobian of the nonlinear function in Eq.(9) at nominal $\hat{z}(t)$ and is given by

$$F(\hat{z}(t)) = \left.\frac{\partial \dot{z}(t)}{\partial z}\right|_{z=\hat{z}(t)} = \begin{bmatrix} A(\hat{\theta}(t)) & D(\hat{z}(t)) \\ [0] & [0] \end{bmatrix} \tag{11}$$

$$D(\hat{z}(t)) = \left.\frac{\partial A(\theta)}{\partial \theta}\right|_{\theta=\hat{\theta}} \hat{x}(t) + \left.\frac{\partial B(\theta)}{\partial \theta}\right|_{\theta=\hat{\theta}} u(t) \tag{12}$$

and we denote $\bar{P} = \bar{P}_k^-$

<u>Update Step:</u>

The posterior estimate of the state is obtained from,

$$\hat{z}_k^+ = \hat{z}_k^- + K_k (y_k - \bar{C}\hat{z}_k^-) \tag{13}$$

where

$$\bar{C} = \begin{bmatrix} C & 0 \end{bmatrix} \tag{14}$$

The Kalman gain K_k and the posterior error covariance $\bar{P}_k^+$ are calculated from

$$K_k = \bar{P}_k^- \bar{C}^T (\bar{C}\bar{P}_k^- \bar{C}^T + R)^{-1} \tag{15}$$

$$\bar{P}_k^+ = (I - K_k \bar{C})\bar{P}_k^- (I - K_k \bar{C})^T + K_k R K_k^T \tag{16}$$

The filter is initialized with

$$\hat{z}_0^+ = E\begin{bmatrix} \hat{x}_0 \\ \hat{\theta}_0 \end{bmatrix} \tag{17}$$

and

$$\bar{P}_0^+ = \begin{bmatrix} P_{x_0} & [0] \\ [0] & P_{\theta_0} \end{bmatrix} \tag{18}$$

Convergence of the augmented filter model requires

$$\left.\frac{\partial K_x(\theta)}{\partial \theta}\right|_{x=\hat{x},\theta=\hat{\theta}} \neq 0 \tag{19}$$

where K_x is the partition of the Kalman gain, corresponding to the un-augmented state, namely

$$K_k \stackrel{def}{=} \begin{bmatrix} K_x \\ K_\theta \end{bmatrix} \tag{20}$$

The augmented state approach requires writing the state space formulation explicitly as a function of the state and parameters in order to calculate the Jacobian $F(\hat{z}(t))$ at each time step, which is easy to implement when small models are taken into consideration. However when the analytical model and the parameter vector is large, that is simply impractical. We present a parameterization scheme in appendix section that allows implementing the EKF-based parameter estimation algorithm efficiently regardless of the size of the model and parameter vector.

2.1 EKF with Fading Memory

Examination shows that the EKF combined state and parameter estimation responds to time variant changes in the parameters very slowly. The reaction time can be improved by using the concept of a fading memory (Fagin 1964). The equations describing the EKF with fading memory are identical to the standard EKF except that an additional step for fading memory is introduced with a forgetting factor matrix after the propagation of state error covariance in Eq.(16), which is

$$\tilde{P}_k^+ = \Lambda \overline{P}_k^+ \Lambda^T \tag{21}$$

where Λ is a diagonal matrix with size (n+p)x(n+p), where n and p represent the numbers of states and parameters, respectively. The first n diagonal elements of Λ are set to 1.0 whereas the next p elements, denoted by λ_1, λ_2,... λ_p are chosen based on how much forgetting of the past data is required. When $\lambda = 1$ there is no fading and in most applications λ is taken only slightly greater than 1 (e.g., $\lambda = 1.001$).

3 NUMERICAL EXPERIMENT: PLANAR TRUSS STRUCTURE

This simulation experiment examines the fictive updating approach for damaged detection using a truss structure. The planar truss structure considered is depicted in Fig.3. All the bars are made of steel (with E = 200 GPa) and have an area of 64.5 cm^2. Mass is 1.75*10^5 kg at each coordinates and damping is taken as 2% in all modes.

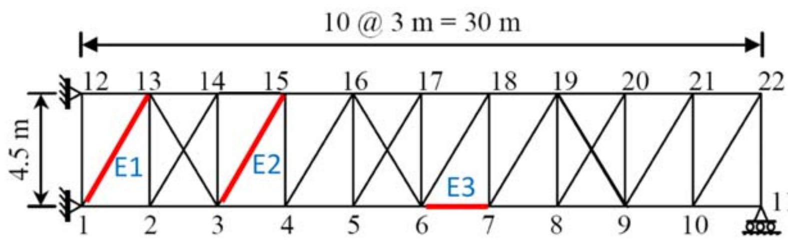

Figure 3: Truss Structure Utilized in the Numerical Testing of the Fictive Updating

We obtain results for two sensor arrangements: (1) Five sensors recording motion in the vertical direction placed at coordinates {2, 4, 6, 8, and 10} (2) Nine sensors recording motion in the vertical direction are located at each of the unsupported joints of the lower chord. The sensor at the coordinate 6 in both arrangements is also recording the horizontal velocity. The deterministic excitation is taken as a segment of white noise process having a unit variance and is applied at coordinate 5. Unmeasured excitations are assumed to act at coordinates {14, 16, and 20} in the vertical and horizontal directions. The deterministic input signal is assumed contaminated by and added noise equal to 10 % of the RMS of the excitation. The output noise is taken to have an RMS equal to 10 % of the RMS of the response measured at the sensor location.

We consider three damage cases defined as 20% followed by 40% loss of stiffness in each of the three bars (one at a time) denoted as E1, E2 and E3. The fictive updating is performed in each case from a simulation of 600 seconds. The fading memory factor of the EKF is fixed as 1.003. The first five un-damped frequencies of the reference model and damaged models for the three cases are depicted in Tables 1-2.

Table 1: First Five Frequencies (Hz) of the Truss Model with 20% Damage Extent in Three Cases

Freq. No	Undamaged Frequency	Bar E1		Bar E2		Bar E3	
		Freq.	%Change	Freq.	%Change	Freq.	%Change
1	0.649	0.642	1.217	0.644	0.870	0.645	0.667
2	1.202	1.202	0.001	1.202	0.006	1.197	0.454
3	1.554	1.523	2.066	1.550	0.281	1.553	0.087
4	2.455	2.379	3.087	2.445	0.411	2.449	0.247

5	3.302	3.293	0.264	3.298	0.121	3.274	0.852

Table 2: First Five Frequencies (Hz) of the Truss Model with 40% Damage Extent in Three Cases

Freq. No	Undamaged Frequency	Bar E1		Bar E2		Bar E3	
		Freq.	%Change	Freq.	%Change	Freq.	%Change
1	0.649	0.629	3.211	0.635	2.279	0.638	1.753
2	1.202	1.202	0.041	1.202	0.017	1.188	1.171
3	1.554	1.472	5.334	1.543	0.745	1.551	0.221
4	2.455	2.287	6.831	2.427	1.129	2.439	0.660
5	3.302	3.283	0.558	3.287	0.436	3.228	2.251

Finite Element Model Based

We use the stiffness of bar E1 as the parameter to be updated. The results are depicted in Fig.4. As can be seen, the update is essentially exact when the damage is actually in bar E1 (as one would expect since the model is exact). When the damage is on bar E2 the result is indicative of the changes, although the update is much smaller than the actual change in the bar and the parameter does not stabilize during the time window when nothing is changing. When damage is on bar E3 the result is poor.

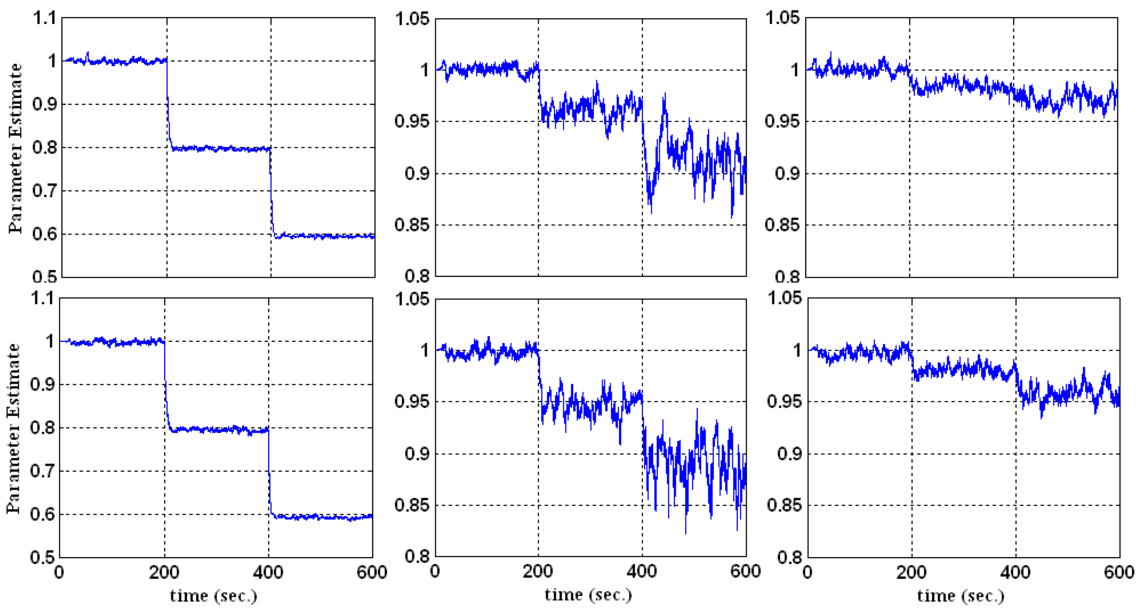

Figure 4: Fictive Parameter Updating of the bar E1 in FE model. Top: Sensor set 1, Bottom: Sensor set 2. Damage Cases; Left: Bar E1, Middle: Bar E2, Right: Bar E3.

Modal Model Based

Another possibility is to track a modal parameter such as frequency. In this case the update is not truly fictive because the frequencies do in fact change as a result of the damage but we retain the term partly for convenience and partly because only one frequency is allowed to change here. In the modal model the matrices are formed using the first 15 pairs of complex modes. The results are depicted in Fig.5.

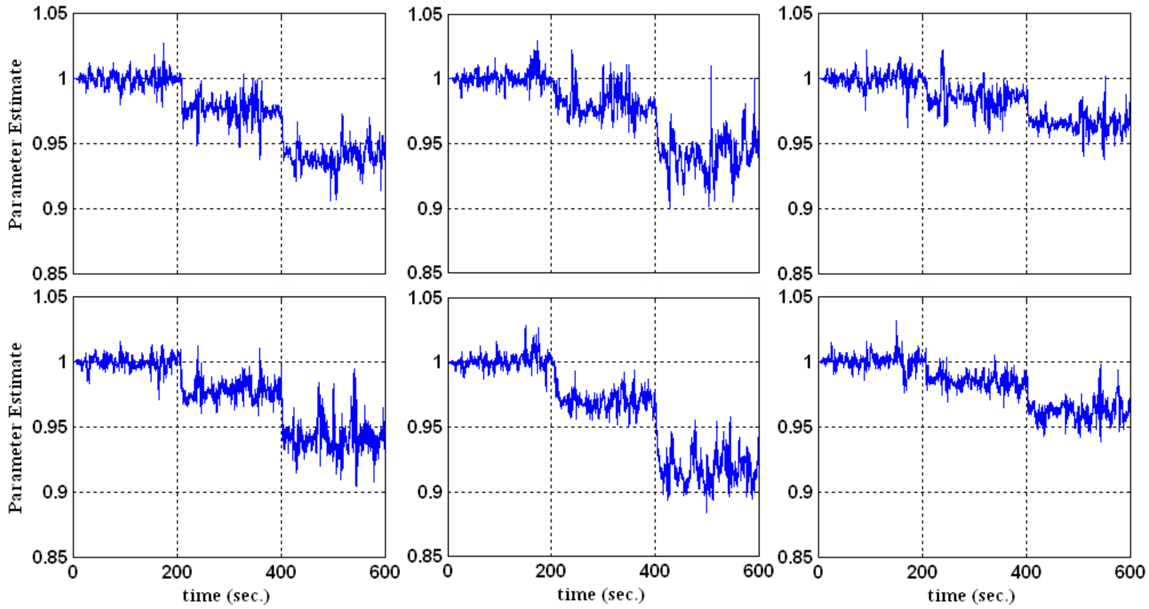

Figure 5: Fictive Updating of the first frequency of the modal model.Top: Sensor set 1, Bottom: Sensor set 2. Damage Cases; Left: Bar E1, Middle: Bar E2, Right: Bar E3.

As can be seen, for the damage case of bar E1, parameter updating lead to 2.5% and 6-5% change in the first frequency, which is larger than the 1.2% and 3.2% real change as seen in Tables 1-2; this is a positive feature. The results for bar E3, however, are less satisfactory. Sensor set 2 lead to improved results compared to set 1, as one would expect since more information is used.

4 CONCLUSIONS

The paper examines the merit of a damage detection strategy based on fictive parameter updating. The objective is the detection of damages that take place abruptly and in situations where the time from the damage appearance to detection is important. The merit of the approach is simplicity but the negative feature detected in this work is the fact that the updated values do not seem to display strong convergence. In the presence of non-stationary coloured input noise one expects that the parameters would tend to fluctuate when there is no change and this will make it more difficult to identify small changes. Further research to determine performance under these conditions is needed before an assessment on merit can be made.

5 APPENDIX

Let the equations of equilibrium of a linear dynamical system be written as

$$M(\theta^m)\ddot{q}(t) + C_\xi(\theta^c)\dot{q}(t) + K(\theta^k)q(t) = b_2 u(t) \tag{22}$$

where the dot represents differentiation with respect to time, $q \in \mathbb{R}^{\eta x1}$ is the displacement vector at the degrees of freedom. M , K and C_ξ are mass, stiffness and damping matrices. θ^m, θ^k and θ^c are finite dimensional parameter vectors which contain parameters related to the mass, stiffness and damping properties of the elements of the model. We define global parameter vector as

$$\theta = \begin{bmatrix} \theta^m & \theta^k & \theta^c \end{bmatrix}^T \tag{23}$$

and parameter vector of the jth member is

$$\theta_j = \begin{bmatrix} \theta_j^{\,m} & \theta_j^{\,k} & \theta_j^{\,c} \end{bmatrix}^T \tag{24}$$

The dynamical system matrices consist of f elements can be parameterized as

$$M(\theta^m) = \sum_{j=1}^{f} \theta_j^m M_j = \theta_1^m M_1 + \theta_1^m M_2 + \cdots + \theta_p^m M_p + \cdots + \theta_f^m M_f \tag{25}$$

$$K(\theta^k) = \sum_{j=1}^{f} \theta_j^k K_j = \theta_1^k K_1 + \theta_2^k K_2 + \cdots + \theta_p^k K_p + \cdots + \theta_f^k K_f \tag{26}$$

$$C_\xi(\theta^c) = \sum_{j=1}^{f} \theta_j^c (C_\xi)_j = \theta_1^c (C_\xi)_1 + \theta_2^c (C_\varsigma)_2 + \cdots + \theta_p^c (C_\xi)_p + \cdots + \theta_j^c (C_\xi)_f \tag{27}$$

where M_j, K_j and $(C_\xi)_j$ denote the stiffness, mass and damping matrices for the jth element in global coordinates and p is the number of the unknown parameters in the structural system matrices. The parameters are in order such that 1 to p refer to unknown parameters and p+1 to f refer to known parameters. These matrices can be partitioned as follows

$$M(\theta^m) = M_P + M_R \tag{28}$$

$$K(\theta^k) = K_P + K_R \tag{29}$$

$$C_\xi(\theta^c) = C_P + C_R \tag{30}$$

where subscript P and R denote the partition due to unknown and known parameters respectively, namely

$$M_P = \sum_{j=1}^{p} \theta_j M_j \quad M_R = \sum_{j=p+1}^{f} \theta_j M_j \tag{31}$$

$$K_P = \sum_{j=1}^{p} \theta_j^k K_j \quad K_R = \sum_{j=p+1}^{f} \theta_j^k K_j \tag{32}$$

$$C_P = \sum_{j=1}^{p} \theta_j^c (C_\xi)_j \quad C_R = \sum_{j=p+1}^{f} \theta_j^c (C_\xi)_j \tag{33}$$

The state space description of the dynamic linear system in Eq.(22) can be written as

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) \tag{34}$$

where $A(\theta)$ and $B(\theta)$ are given in the form,

(35)

$$A(\theta) = \begin{bmatrix} 0 & I \\ -M(\theta^m)^{-1}K(\theta^k) & -M(\theta^m)^{-1}C_\xi(\theta^c) \end{bmatrix} \tag{36}$$

$$B(\theta) = \begin{bmatrix} 0 \\ M(\theta^m)^{-1} b_2 \end{bmatrix} \tag{37}$$

Using definitions given in Eqs.(28)-(30), one can partition $A(\theta)$ and $B(\theta)$ in Eq.(34) due to unknown and known parameters as follows,

$$A(\theta) = A_R + A_P \tag{38}$$

$$B(\theta) = B_R + B_P \tag{39}$$

where

$$A_R = \begin{bmatrix} 0 & I \\ -M_R^{-1}K_R & -M_R^{-1}(C_\xi)_R \end{bmatrix} \quad (40)$$

$$A_P = \begin{bmatrix} 0 & 0 \\ -\frac{1}{\theta_1^m}\theta_1^k M_1^{-1}K_1 & -\frac{1}{\theta_1^m}\theta_1^k M_1^{-1}(C_\xi)_1 \end{bmatrix} + \cdots + \begin{bmatrix} 0 & 0 \\ -\frac{1}{\theta_p^m}\theta_1^k M_1^{-1}K_1 & -\frac{1}{\theta_1^m}\theta_p^k M_p^{-1}(C_\xi)_p \end{bmatrix} \quad (41)$$

$$B_R = \begin{bmatrix} 0 \\ -M_R^{-1}b_2 \end{bmatrix}, \quad B_P = \begin{bmatrix} 0 \\ -\frac{1}{\theta_p^m}M_1^{-1}b_2 \end{bmatrix} + \cdots + \begin{bmatrix} 0 \\ -\frac{1}{\theta_p^m}M_p^{-1}b_2 \end{bmatrix} \quad (42)$$

Now we recall the Jacobian of the augmented state space model as follows

$$F(\hat{z}(t)) = \left.\frac{\partial(\dot{z}(t))}{\partial\theta}\right|_{z=\hat{z}(t)} = \begin{bmatrix} A(\hat{\theta}(t)) & D^m(\hat{\theta}(t)) & D^k(\hat{\theta}(t)) & D^c(\hat{\theta}(t)) \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (43)$$

where

$$D^m(\hat{\theta}(t)) \overset{def}{=} \left.\frac{\partial(A(\theta))\hat{x}}{\partial\theta^m}\right|_{\theta=\hat{\theta}} + \left.\frac{\partial(B(\theta))u(t)}{\partial\theta^m}\right|_{\theta=\hat{\theta}} = \begin{bmatrix} d_1^m & \dots & d_p^m \end{bmatrix} \quad (44)$$

$$D^k(\hat{\theta}(t)) \overset{def}{=} \left.\frac{\partial(A(\theta))\hat{x}}{\partial\theta^k}\right|_{\theta=\hat{\theta}} = \begin{bmatrix} d_1^k & \dots & d_p^k \end{bmatrix} \quad (45)$$

$$D^c(\hat{\theta}(t)) \overset{def}{=} \left.\frac{\partial(A(\theta))\hat{x}}{\partial\theta^c}\right|_{\theta=\hat{\theta}} = \begin{bmatrix} d_1^c & \dots & d_p^c \end{bmatrix} \quad (46)$$

and

$$d_j^m = \begin{bmatrix} 0 & 0 \\ \frac{1}{(\hat{\theta}_j^m)^2}\hat{\theta}_j^k M_j^{-1}K_j & \frac{1}{(\hat{\theta}_j^m)^2}\hat{\theta}_j^c M_j^{-1}(C_\xi)_j \end{bmatrix}\hat{x} + \begin{bmatrix} 0 \\ \frac{1}{(\hat{\theta}_j^m)^2}M_j^{-1}b_2 \end{bmatrix}u(t) \quad (47)$$

$$d_j^k = \begin{bmatrix} 0 & 0 \\ -\frac{1}{\hat{\theta}_j^m}M_j^{-1}K_j & 0 \end{bmatrix}\hat{x} \quad (48)$$

$$d_j^c = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{\hat{\theta}_j^m}M_j^{-1}(C_\xi)_j \end{bmatrix}\hat{x} \quad (49)$$

d_j is a column vector with the size nx1 where we recall that n is order of the system. The size of Jacobian matrix, $F(\hat{z}(t))$ is (n+3p)(n+3p). It's apparent that the calculation of $F(\hat{z}(t))$ requires only the dynamical matrices of the elements in global coordinates that parameters are being updated, namely M_j, K_j and $(C_\xi)_j$. After one have the a priori estimate of the augmented state, the d_j's for each parameter can be calculated from Eqs. (47)-(49), then Jacobian matrix $F(\hat{z}(t))$ is constructed from Eq.(43).

REFERENCES

Beard, R. V. 1971. *Failure accommodation in linear system through self reorganization*. Ph.D. thesis. Massachussets Institute of Technology.

Fagin, S.L. 1964. Recursive linear regression theory, optimal filter theory and error analysis optimal system. *IEEE Int. Convention Record*, p. 216-240.

Fritzen, C.P., Seibold S. and Buchen, D. 1995. Application of Filter Techniques for Damage Identification in Linear and Nonlinear Mechanical Structures. *Proc. of the 13th International Modal Analysis Conference*, p. 1874-1881

Jones, H. L. 1973. *Failure detection in linear systems*. Ph.D. thesis. Massachussets Institute of Technology.

Liu, X., Escamilla-Ambrosio, P.J. and Lieven, N.A.J. 2009. Extended Kalman filtering for the detection of damage in linear mechanical structures. *Journal of Sound and Vibration*, p. 1023-1046

Mehra, R. K. and Peshon, I. 1971. An innovations approach to fault detection and diagnosis in dynamic systems. *Automatica*, p. 637-640

Kopp, R. E. and Orford, R. J. 1963. Linear Regression Applied to System Identification for Adaptive Control Systems, *AIAA Journal*, p. 2300-2306.

Ljung L., Asymptotic behavior of the extended Kalman filter as a parameter estimator for linear systems, IEEE Transactions on Automatic Control, (1979), pp. 36 - 50

Panuska, V., A new form of the extended Kalman filter for parameter estimation in linear systems, Decision and Control including the Symposium on Adaptive Processes (1979), pp. 927 -932

Panuska, V., A new form of the extended Kalman filter for parameter estimation in linear systems with correlated noise, Automatic Control, IEEE Transactions on (1980), pp. 229 – 235.

Yang, J. N., Lin, S. , Huang, H. and Zhou, L. 2005. An adaptive extended Kalman filter for structural damage identification, *Structural Control and Health Monitoring*, p. 849-867.

Walter, E. and Lecourtier, Y., Global approaches to identifiability testing for linear and nonlinear state space models, Mathematics and Computers in Simulation (1982), pp. 472 - 482